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成果の概要／山川 誠

近年，制震構造と呼ばれる建築構造物へ制震装置を利用するシステムについての研究・開発が広範囲にわたって展開されている．制震装置を利用した減衰機構の付加により構造物の基本構造を大きく変えることなく耐震安全性を高めることができるが，これまでの耐震設計法とは基本的な設計思想が異なり，経験と勘に頼った設計では対応が難しくなっている．このような背景の下で，建築構造物における主体架構の部材断面および制震装置の配置に関する合理的な設計法の確立が要望されている．

世界地震工学会議は報告者の専門とする建築構造のみならず，地震工学に関連する多くの分野の国際的な研究者および技術者が研究発表および意見交換を行う場であり，今回も 3000 名以上の参加があった．世界地震工学会議の研究発表セッションに参加することにより，提案する設計法に関する学術的および実務的な観点両面からの有意義な意見交換を行うこと，さらに国際的に高い水準にある他の関連研究に関する調査を通じて，提案手法をより発展させるための情報を得ることを目的として貴財団の国際研究集会派遣助成を受け，第 14 回世界地震工学会議に出席した．

制震構造の最適設計に関する既往の研究には設計用応答スペクトルやエネルギースペクトルのような簡便な方法による応答制約問題を扱うものが多い．これらの方法では必要となる計算量が少なく，さらに得られる設計解の性質を理解することも容易である．しかし，実務構造設計では数波の設計用地震動に対する詳細な非線形時刻歴応答解析に基づく検証解析を行うことが一般的である．そのため，既往の最適設計に関する研究成果を実務設計に直接利用することは難しい．設計の合理化により社会貢献を目指す立場においては，計算負荷や解の収束性を含めて解決を要する課題が未だ多く存在すると言える．

数理統計学の分野では(1)計算機への入力と出力に着目してある解析結果を補正して他の解析結果を統計的に予測する方法(2)詳細解析と簡易解析の両方を組み合わせて近似最適化を行う可変複合モデルが提案されている．これらの方法では計算機実験に基づく数理統計的手法による補正モデルの有効性が示されている．本研究ではこのような統計的な予測モデルを用いることにより，実務的な解析法，最適設計に適した解析法等の複数の解析法・解析モデルを統合化可能な合理的な設計法を提案した．本研究で対象とする設計問題は以下のように記述される．

設計問題

速度依存型の制震装置が付加された鋼構造建築骨組において，与えられた設計用地震動に対する最大層間変形角，部材許容応力度を指定値以下とする制約条件を満足し，コストが最小となるような柱・梁の部材断面および各層の制振装置の配置量を求めよ．

ここで，柱・梁の部材断面の大きさを表す変数として部材断面積を，制振装置の配置量を表す変数としてダンパー耐力を選ぶ．この設計問題を解くために提案した設計法の概要は以下の通りである．

(Step1) 応答スペクトル法による簡便な解析法を用いた最適設計問題を解いて，初期解を得る．

(Step2) 設計領域内での計算機実験を行い，簡便な解析と検証用の詳細な解析（時刻歴応答解析法）のそれぞれにおける地震時応答のデータを得る．

(Step3) 得られた地震時応答のデータに対して，交差検証法と呼ばれる方法を用いて統計的予測に必要なパラメータを決定する．

(Step4) ベイズ推定と呼ばれる統計的予測法を用いた最適設計問題を解いて，最適解の予測値を得る．

(Step5) 詳細な解析を用いて，設計解の地震時応答の検証を行う．予測精度が十分でなければ，この設計解の地震時応答をデータに追加して(Step3)に戻る．

図 1,2 に示す 10 層 3 スパンの鋼構造平面モデルに提案した設計法を適用し，最適な部材断面と付加する制振装置の各層配置量を求めた．応答予測に基づき得られた最適解を表 1 に示す．得られた最適解に対して，詳細な解析法（時刻歴応答解析法）により地震時応答の検証を行った．図 3 に，地震時応答（各層最大層間変形角）の指定上限値，予測範囲，検証値をそれぞれ示す．図 3 より，地震時応答（各層最大層間変形角）の検証値は予測範囲内にほぼ収まり，指定上限値もほぼ満足していることが示されている．

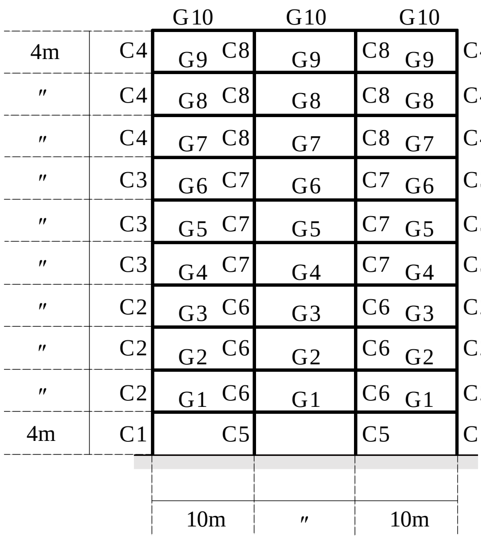

図 1　10 層 3 スパンモデル

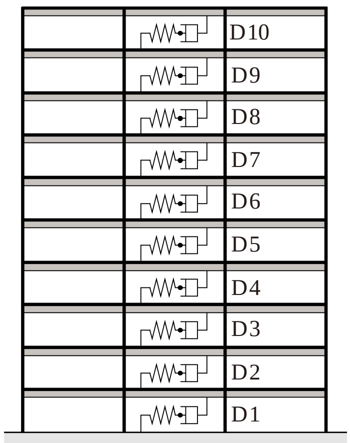

図 2　制震装置の解析モデル

表 1　応答予測による最適解

柱断面積 (cm²)		梁断面積 (cm²)		各層制震装置耐力 (kN)	
C1	254.7	G1	256.1	D1	70.0
C2	254.7	G2	321.6	D2	100.0
C3	209.6	G3	244.8	D3	100.0
C4	200.0	G4	310.4	D4	100.0
C5	370.7	G5	250.3	D5	100.0
C6	370.7	G6	247.1	D6	87.1
C7	352.7	G7	231.3	D7	83.9
C8	316.6	G8	201.8	D8	100.0
		G9	162.4	D9	70.0
		G10	118.2	D10	70.0

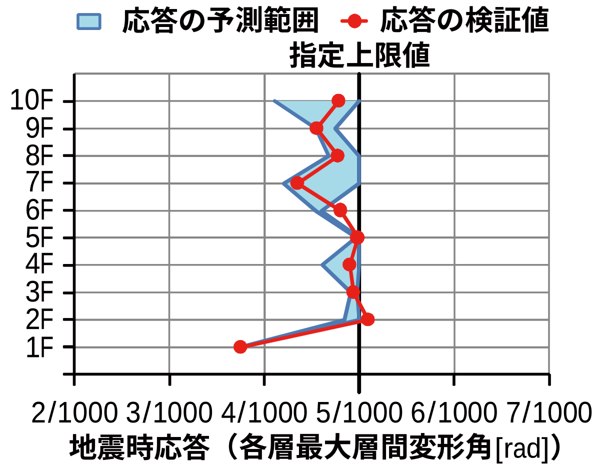

図 3　応答の予測精度

本研究は以下の学術的に新規な提案および知見を含む.

(1) 計算機実験に基づいた補正モデルを用いることにより，応答スペクトル法のような簡便な方法と同等の計算量で，非線形時刻歴応答解析のような詳細な解析法の応答値を統計的に予測する方法を提案した.

(2) これまでに提案された補正モデルを拡張し，正則化項と等価な誤差項を付加して不確定性を考慮することにより，既往の方法よりも予測パラメータの不確定性に関して頑強性の高い方法を提案した.

(3) 数値解析例により，本提案法により応答の予測が精度良く行えることを確認し，設計解が効率的に得られることを示した.

本研究成果は合理的な制震構造の設計法の確立へ繫がるものである．本国際研究集会への参加により，研究を進めるために有益な意見交換および情報を得ることができた．本研究課題の遂行を通じて得られた成果を基に，今後も学術的，社会的両面にわたる貢献を目的とした研究の継続を予定している．今回の国際研究集会派遣助成事業による貴財団の援助に謝意を表する．

Optimum Design Method of Viscous Dampers in Building Frames Using Calibration Model

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ABSTRACT :

We present a new practical optimum design method of viscous dampers in building frames. In the design field of structural engineering, structural designers always try to build mathematical models to simulate the exact behavior of real world system which are often very complex for the verification analysis and are not suited for the optimum design owing to the fact that they come with a good deal of computational cost. Therefore, we present a method using calibrated response model, which is simpler than the verification model. The calibration model is a statistical prediction of the verification model and with its usage the optimum design method of dampers can be made efficient and numerically stable. The efficiency of the presented method has been demonstrated by a numerical example.

KEYWORDS : Optimum design, Viscous damper, Calibration, Bayesian inference

1. INTRODUCTION

Structural designers, in structural design, always try to develop mathematical models which can accurately simulate the behavior of the real world system. However, these models for the verification analysis are often complex and are not suited for the optimum design owing to the face that they come with a good deal of computational cost. Furthermore, it is difficult to understand the characteristics of the optimum solution. For this reason some studies of an optimum design method of viscous dampers [for example, Tsuji et al., 2000] in building frames have dealt with simpler model than the model widely used for the verification analysis. Here, we present a new practical optimum design method of viscous dampers in building frames using statistical prediction model within Bayesian framework. The presented method is based on the calibrated output of the simple model and is very efficient in view of computational cost and numerically stability.

1.1. Multi-level Models

Kennedy and O'Hagan (2001) modeled the outputs of multi-level computer codes using a spatial autocorrelation structure within Bayesian framework. In this paper, we use this multi-level model: verification model and simple model. We define a model using nonlinear time history response analysis as verification model and a model using extended CQC method [Igusa et al., 1984] as simple model. The calibration model is defined as the scaled simple model based on computer experiments. The response by verification model is predicted as the scaled simple mode. The method is very efficient and numerically stable. Using this method, we formulate the optimum design problem, which finds a set of viscous dampers, the cross sections of columns and beams that minimize the cost function subject to some constraints.

2. CALIBRATION MODEL

From a Bayesian perspective, uncertainty about the output of complex model can be expressed by a stochastic process. The prediction method is assumed to be a function of a set of inputs denoted by $\boldsymbol{x} = (x_1, \cdots, x_{n_X}) \subset \mathbb{R}^{n_X}$, with output of complex model represented by $y \in \mathbb{R}$. Let us consider a model:

$$y(\boldsymbol{x}) = \eta(\boldsymbol{x}) + \varepsilon, \tag{2.1}$$

where ε is a random noise and follows an independent normal distribution: $\varepsilon \sim \mathcal{N}(0, \alpha\sigma^2/(1+\alpha))$. The parameters σ^2 and α is unknown scale parameter and variance ratio. Such models typically include an unknown smooth response surface, which is knows as *"nugget effect"*.

2.1. Gaussian Process

We assume that $\eta(\boldsymbol{x})$ is represented by Gaussian process [for example, Rasmussen et al., 2006] with mean $m_0(\boldsymbol{x})$ and covariance function $V_0(\boldsymbol{x}, \boldsymbol{x}')$. Gaussian process using a hierarchical formulation is expressed as

$$\eta(\boldsymbol{x}) \mid \boldsymbol{\beta}, \sigma^2, \psi \sim \mathcal{GP}\left(m_0(\boldsymbol{x}), V_0\left(\boldsymbol{x}, \boldsymbol{x}'\right)\right), \tag{2.2}$$

where $\mathcal{GP}(\cdot,\cdot)$ denotes Gaussian process distribution, and

$$m_0(\boldsymbol{x}) = \boldsymbol{h}(\boldsymbol{x})^T \boldsymbol{\beta}, \tag{2.3}$$

$$V_0\left(\boldsymbol{x}, \boldsymbol{x}'\right) = \sigma^2 R\left(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{\psi}\right)/(1+\alpha), \tag{2.4}$$

where $\boldsymbol{h}(\boldsymbol{x}) : \mathbb{R}^{n_X} \to \mathbb{R}^q$ is a function of the inputs $\boldsymbol{x}$, which is the simple model. The parameter $\boldsymbol{\beta} \in \mathbb{R}^q$ is an unknown vector of coefficients, and $R(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{\psi})$ is a given correlation function. In this paper, we use Gaussian correlation function as follows

$$R\left(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{\psi}\right) = \exp\left[-\sum_{k=1}^{q}\left(h_k(\boldsymbol{x}) - h_k\left(\boldsymbol{x}'\right)\right)^2 / \psi_k\right], \tag{2.5}$$

where the hyper parameter $\boldsymbol{\psi} = \{\psi_1, \cdots, \psi_q\}$ are called as correlation length parameters. We have a training dataset $\mathcal{D} = \{\boldsymbol{x}^i \in \mathbb{R}^{n_X}, y_i = y(\boldsymbol{x}^i); i = 1, \cdots, n\}$. According to (2.2), the distribution of $\boldsymbol{\eta} = (\eta(\boldsymbol{x}^1), \cdots, \eta(\boldsymbol{x}^n))$ is multivariate normal as follows

$$\boldsymbol{\eta} \mid \boldsymbol{\beta}, \sigma^2, \boldsymbol{\psi} \sim \mathcal{N}\left(\boldsymbol{H}\boldsymbol{\beta}, \sigma^2/(1+\alpha)\boldsymbol{R}_0\right), \tag{2.6}$$

where

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{h}\left(\boldsymbol{x}^1\right)^T \\ \vdots \\ \boldsymbol{h}\left(\boldsymbol{x}^n\right)^T \end{bmatrix}, \quad \boldsymbol{R}_0 = \begin{bmatrix} R\left(\boldsymbol{x}^1, \boldsymbol{x}^1; \boldsymbol{\psi}\right) & \cdots & R\left(\boldsymbol{x}^1, \boldsymbol{x}^n; \boldsymbol{\psi}\right) \\ \vdots & \cdots & \vdots \\ R\left(\boldsymbol{x}^n, \boldsymbol{x}^1; \boldsymbol{\psi}\right) & \cdots & R\left(\boldsymbol{x}^n, \boldsymbol{x}^n; \boldsymbol{\psi}\right) \end{bmatrix}. \tag{2.7),(2.8}$$

Using standard techniques for conditioning in multivariate normal distributions, we get Gaussian process prediction with mean $m_0^*(\boldsymbol{x})$ and covariance function $V_0^*(\boldsymbol{x}, \boldsymbol{x}')$ as

$$\eta(\boldsymbol{x}) \mid \alpha, \boldsymbol{\beta}, \sigma^2, \boldsymbol{\psi}, \boldsymbol{y} \sim \mathcal{GP}\left(m_0^*(\boldsymbol{x}), V_0^*\left(\boldsymbol{x}, \boldsymbol{x}'\right)\right), \tag{2.9}$$

where

$$m_0^*(\boldsymbol{x}) = \boldsymbol{h}(\boldsymbol{x})^T \boldsymbol{\beta} + \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1}(\boldsymbol{y} - \boldsymbol{H}\boldsymbol{\beta}), \tag{2.10}$$

$$V_0^*\left(\boldsymbol{x}, \boldsymbol{x}'\right) = \sigma^2\left\{R\left(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{\psi}\right)/(1+\alpha) - \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1}\boldsymbol{r}\left(\boldsymbol{x}'\right)\right\}, \tag{2.11}$$

$$\boldsymbol{R} = (\boldsymbol{R}_0 + \alpha \boldsymbol{I}_n)/(1+\alpha), \tag{2.12}$$

$$\boldsymbol{r}(\boldsymbol{x}) = \left(R(\boldsymbol{x}, \boldsymbol{x}_1; \boldsymbol{\psi})/(1+\alpha) \quad \cdots \quad R(\boldsymbol{x}, \boldsymbol{x}_n; \boldsymbol{\psi})/(1+\alpha) \right)^T. \tag{2.13}$$

where $\boldsymbol{I}_n$ denotes the identity matrix of size n. The model is also known as "*kriging*". *Kriging* is mostly used in two or three dimensional input spaces for spatial prediction, although Gaussian process prediction could be used in a general regression context.

2.2. Gaussian Process within Bayesian Framework

Using a weak prior for $p(\boldsymbol{\beta}, \sigma^2) \propto \sigma^{-2}$, integrating out $\boldsymbol{\beta}$ and σ^2 of (2.9) by Bayes' theorem, the Student's *t* process predictor with $n-q$ degrees of freedom, mean $m(\boldsymbol{x})$ and covariance function $V(\boldsymbol{x}, \boldsymbol{x}')$ can be shown as

$$\eta(\boldsymbol{x}) \mid \boldsymbol{y}, \alpha, \psi \sim \mathcal{TP}\left(n-q, m(\boldsymbol{x}), V(\boldsymbol{x}, \boldsymbol{x}')\right) \tag{2.14}$$

where $\mathcal{TP}(\cdot,\cdot,\cdot)$ denotes Student's *t* process distribution, and

$$m(\boldsymbol{x}) = \boldsymbol{h}(\boldsymbol{x})^T \hat{\boldsymbol{\beta}} + \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1} \left(\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{\beta}} \right), \tag{2.15}$$

$$\begin{aligned} V(\boldsymbol{x}, \boldsymbol{x}') = \hat{\sigma}^2 \Big[& C(\boldsymbol{x}, \boldsymbol{x}'; \psi)/(1+\alpha) - \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1} \boldsymbol{r}(\boldsymbol{x}') \\ & + \left(\boldsymbol{h}(\boldsymbol{x})^T - \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1} \boldsymbol{H} \right) \left(\boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{H} \right)^{-1} \left(\boldsymbol{h}(\boldsymbol{x}') - \boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{r}(\boldsymbol{x}') \right) \Big], \end{aligned} \tag{2.16}$$

$$\hat{\boldsymbol{\beta}} = \left(\boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{H} \right)^{-1} \boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{y}, \tag{2.17}$$

$$\hat{\sigma}^2 = \boldsymbol{y}^T \left(\boldsymbol{R}^{-1} - \boldsymbol{R}^{-1} \boldsymbol{H} \left(\boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{H} \right)^{-1} \boldsymbol{H}^T \boldsymbol{R}^{-1} \right) \boldsymbol{y} / (n-q-2). \tag{2.18}$$

According to (2.1) and (2.14), conditional expectation and variance are obtained as follows:

$$\mathbb{E}[y(\boldsymbol{x}) \mid \boldsymbol{y}, \alpha, \boldsymbol{\psi}] = \boldsymbol{h}(\boldsymbol{x})^T \hat{\boldsymbol{\beta}} + \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1} \left(\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{\beta}} \right), \tag{2.19}$$

$$\begin{aligned} \mathbb{V}[y(\boldsymbol{x}) \mid \boldsymbol{y}, \alpha, \boldsymbol{\psi}] = \hat{\sigma}^2 \Big[& 1 - \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1} \boldsymbol{r}(\boldsymbol{x}) \\ & + \left(\boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{r}(\boldsymbol{x}) \right)^T \left(\boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{H} \right)^{-1} \left(\boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{r}(\boldsymbol{x}) \right) \Big]. \end{aligned} \tag{2.20}$$

3. OPTIMUM DESIGN METHOD

We present an optimum design method which enables to find minimum cost so as to satisfy member-end strain constraints, story drift constraints, and side constraints. Consider a planar steel building frame added Maxwell-type viscous dampers as shown in Figure 1. The nodal mass is taken into account in every node. Rigid diaphragms are assumed for floor slabs and the dampers connect these rigid diaphragms.

3.1. Design Variable

The cross sectional area of the ith member A_i is associated with the design variables $\boldsymbol{x} = \{x_l; l = 1, \cdots, n_X\}$ as $A_i = x_l \ (i \in \mathcal{I}_{Al})$ with the index set $\mathcal{I}_{Al}$. We simply write it as $A_i(\boldsymbol{x})$. The cross sectional area $A_i(\boldsymbol{x})$, second moment area $I_i(\boldsymbol{x})$ and section modulus $Z_i(\boldsymbol{x})$ are assumed to satisfy:

for beams: $I_i(\boldsymbol{x}) = 4.0 \left(A_i(\boldsymbol{x}) \right)^2, Z_i(\boldsymbol{x}) = 1.5 \left(A_i(\boldsymbol{x}) \right)^{1.5} \quad (i = 1, \cdots, n_M),$ (3.1),(3.2)

for columns: $I_i(\boldsymbol{x}) = 1.2 \left(A_i(\boldsymbol{x}) \right)^2, Z_i(\boldsymbol{x}) = 0.8 \left(A_i(\boldsymbol{x}) \right)^{1.5} \quad (i = 1, \cdots, n_M),$ (3.3),(3.4) where n_M denotes the number of members. Maxwell-type bilinear viscous elements which depend on velocity such as oil damper are added to the frame shown in Figure 1. The jth damper has bilinear damping coefficient: c_{Dj} and $0.1 \times c_{Dj}$, relief load f_{Rj}, maximum load f_{Dj} as shown in Figure 2 and the stiffness $k_{Dj}(\boldsymbol{x})$. The f_{Dj} is also associated with $\boldsymbol{x}$ as $f_{Dj} = x_l \ (j \in \mathcal{I}_{Dl})$ with the index set $\mathcal{I}_{Dl}$, which we simply write as $f_{Dj}(\boldsymbol{x})$. The damping coefficient $c_{Dj}(\boldsymbol{x})$ and stiffness $k_{Dj}(\boldsymbol{x})$ are given as follows:

$$c_{Dj}(\boldsymbol{x}) = (1/4.38) f_{Dj}(\boldsymbol{x}), \quad k_{Dj}(\boldsymbol{x}) = 2.0 f_{Dj}(\boldsymbol{x}) \quad (j = 1, \cdots, n_F) \tag{3.5),(3.6}$$

where n_F denotes the number of stories, and the units of $f_{Dj}(\boldsymbol{x}), c_{Dj}(\boldsymbol{x})$ and $k_{Dj}(\boldsymbol{x})$ are $\mathrm{kN}, \mathrm{kN/(cm/sec)}$ and $\mathrm{kN/cm}$, respectively.

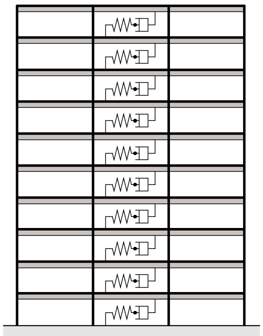

Figure 1 Planar steel frame added Maxwell-type viscous dampers

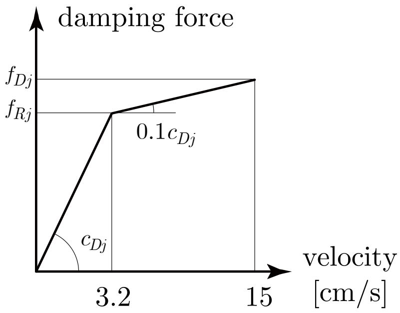

Figure 2 Damping characteristic of damper

3.2. *Verification Model*

We perform the nonlinear time history response analysis by Newmark-β method as the verification model. The design earthquake motions and the amplitude of the scaled design earthquakes are as shown in Table 1.

Table 1 Design earthquakes (by Building Center of Japan)

	Maximum amplitude	
	seismic velocity (cm/s)	seismic acceleration (cm/s^2)
EL CENTRO 1940 NS	25	255
TAFT 1952 EW	25	248
BCJ-L1 (artificial seismic wave)	29	207

Let $\delta_{j\max}^{V}(\boldsymbol{x})$ denote the maximum story drift of the jth story to the design earthquakes by time history response analysis. In this model, the dampers are bilinear viscous elements, however, the columns and the beams are elastic elements for simplicity. We suppose that the nonlinear elasto-plastic elements can be used.

3.3. *Simple Model*

We use the extended CQC method as simple model. Let ω and h denote an eigen frequency and a modal damping ratio. Design response spectrum [Newmark and Hall, 1982] is modified for the design earthquakes shown in Table 1. The following design displacement response spectrum is used here.

$$S_D(\omega, h) = \min\left[S_D^A(\omega, \mathrm{h}), S_D^V(\omega, \mathrm{h})\right] \tag{3.7}$$

$$S_D^A(\omega, \mathrm{h}) = 261.1\{4.42 - 1.00\ln(100\mathrm{h})\}/\omega^2 \ \mathrm{(cm)} \tag{3.8}$$

$$S_D^V(\omega, \mathrm{h}) = 32.4\{2.62 - 0.51\ln(100\mathrm{h})\}/\omega \ \mathrm{(cm)} \tag{3.9}$$

Let $\delta^{S}_{j\max}(\boldsymbol{x})$ denote the maximum story drift to the design response spectrum by extended CQC method. Note that, in this simple model, the jth damper is linear viscous element which has the initial damping coefficient of the verification model c_{Dj}. Additionally, we also use output of CQC method with no damper model, that is, $c_{Dj}=0$ as simple model. Let $\delta^{S0}_{j\max}(\boldsymbol{x})$ denote the maximum story drift to the design response spectrum by CQC method in the no damper model. $\delta^{V}_{j\max}(\boldsymbol{x})$ will be in between $\delta^{S}_{j\max}(\boldsymbol{x})$ and $\delta^{S0}_{j\max}(\boldsymbol{x})$.

3.4. Original Optimum Design Problem

The cost function of the frame model is given as

$$f(\boldsymbol{x}) = \sum_{i=1}^{n_M} w_F \rho l_i A_i(\boldsymbol{x}) + \sum_{j=1}^{n_F} w_D f_{Dj}(\boldsymbol{x}) \tag{3.10}$$

wehre l_i, ρ, w_F and w_D are the length of the ith member, the density of steel, the cost factor of frame and damper, respectively. We define the *Original Optimum Design Problem* (*OODP*) as follows:

<u>*OODP*</u>

$$\min_{\boldsymbol{x}} \quad f(\boldsymbol{x}) = \sum_{i=1}^{n_M} w_F \rho l_i A_i(\boldsymbol{x}) + \sum_{j=1}^{n_F} w_D f_{Dj}(\boldsymbol{x}) \tag{3.11}$$

$$\text{subject to} \quad -\bar{\delta}_{j\max} \le \delta^{V}_{j\max}(\boldsymbol{x}) \le \bar{\delta}_{j\max} \quad (j=1,\cdots,n_F) \tag{3.12}$$

$$-\bar{\varepsilon}_{\max} \le \varepsilon_{i\max}(\boldsymbol{x}) \le \bar{\varepsilon}_{\max} \quad (j=1,\cdots,n_F) \tag{3.13}$$

$$\bar{x}^{L}_{l} \le x_l \le \bar{x}^{U}_{l} \quad (l=1,\cdots,n_X) \tag{3.14}$$

where $\varepsilon_{i\max}(\boldsymbol{x})$ denotes the maximum member-end strain of the ith member. $\bar{\delta}_{j\max}$, $\bar{\varepsilon}_{\max}$, $\bar{x}^{L}_{l}$ and $\bar{x}^{U}_{l}$ denote the maximum story drift, the maximum member-end strain, lower and upper bound of the design variable, respectively. For simplicity, we use extended CQC method for $\varepsilon_{i\max}(\boldsymbol{x})$.

3.5. Solution Algorithm with Bayesian Inference

Time history response analysis needs much computational cost than extended CQC method. For this reason, the directly solving *OODP* is expensive in terms of the computer time. Besides, the time history response analysis is strongly nonlinear and non-convex, hence solving *OODP* is not numerically stable. Therefore, we present an optimization method with the statistical prediction model instead of solving *OODP* directly. The solution algorithm is shown as follows:

Step 1. Solving the *Simple Optimum Design Problem*

Firstly, we solve the following *Simple Optimum Design Problem* (*SODP*).

<u>*SODP*</u>

$$\min_{\boldsymbol{x}} \quad f(\boldsymbol{x}) = \sum_{i=1}^{n_M} w_F \rho l_i A_i(\boldsymbol{x}) + \sum_{j=1}^{n_F} w_D f_{Dj}(\boldsymbol{x}) \tag{3.15}$$

$$\text{subject to} \quad -\bar{\delta}_{j\max} \le \delta^{S}_{j\max}(\boldsymbol{x}) \le \bar{\delta}_{j\max} \quad (j=1,\cdots,n_F) \tag{3.16}$$

$$-\bar{\varepsilon}_{\max} \le \varepsilon_{i\max}(\boldsymbol{x}) \le \bar{\varepsilon}_{\max} \quad (i=1,\cdots,n_M) \tag{3.17}$$

$$\bar{x}^{L}_{l} \le x_l \le \bar{x}^{U}_{l} \quad (l=1,\cdots,n_X) \tag{3.18}$$

The solution of the *SODP* is denoted by $\hat{\boldsymbol{x}}^0 = \{\hat{x}^0_l; l=1,\cdots,n_X\}$. Extended CQC method is modal analysis for linear model, as a result solving *SODP* needs low computational cost.

Step 2. Carrying out computer experiments

We carry out the computer analyses using both the simple model and the verification model at the set of the points $\{\boldsymbol{x}^i; i=1,\cdots,n\}$. The points are chosen to be satisfied

$$\overline{x}_l^L \le x_l^i \le \overline{x}_l^U \quad (i=1,\cdots,n,\, l=1,\cdots,n_X). \tag{3.19),(3.20}$$

$$\hat{x}_l^0 - \Delta\overline{r}_l \le x_l^i \le \hat{x}_l^0 - \Delta\overline{r} \quad (i=1,\cdots,n,\, l=1,\cdots,n_X). \tag{3.21),(3.22}$$

where x_l^i and $\Delta\overline{r}_l$ denote the l th element of $\boldsymbol{x}^i$ and change of the l th design variable, respectively. Here, we set the equations presented in section 2 as follows:

$$y^{(j)}(\boldsymbol{x}) = \delta_{j\max}^V(\boldsymbol{x}) \quad (j=1,\cdots,n_F), \tag{3.23}$$

$$\boldsymbol{h}(\boldsymbol{x}) = \left(\delta_{1\max}^S(\boldsymbol{x}),\cdots,\delta_{n_F\max}^S(\boldsymbol{x}),\delta_{1\max}^{S0}(\boldsymbol{x}),\cdots,\delta_{n_F\max}^{S0}(\boldsymbol{x})\right)^T. \tag{3.24}$$

In equation (3.23), the method in section 2 is applied for each $j=1,\cdots,n_F$. Thus, we get the training dataset of j th story $\mathcal{D}_j = \{\boldsymbol{x}^i, y_i^{(j)}; i=1,\cdots,n\}$. Various methods of choosing points are presented [for example, Santner, et al., 2003]. Here, the points are generated randomly for simplicity.

Step 3. Finding the hyper parameters by cross validation

We find the optimum hyper parameters of each $y^{(j)}(\boldsymbol{x})$ by minimizing leave-one-out cross-validated sum of squared error as follows:

$$\hat{\alpha}^{(j)}, \hat{\boldsymbol{\psi}}^{(j)} = \underset{\alpha^{(j)},\boldsymbol{\psi}^{(j)}}{\arg\min} \sum_{k=1}^{n} \sum_{l\in\{\mathcal{I}_k\}} \left\{ y_l^{(j)} - \mathbb{E}\left[y(\boldsymbol{x}_l) \mid \boldsymbol{y}_{\mathcal{I}_k}^{(j)}, \alpha^{(j)}, \boldsymbol{\psi}^{(j)} \right] \right\}^2, \tag{3.25}$$

where $\mathcal{I}_k = \{1,\cdots,n\} \setminus \{k\}$ and $\boldsymbol{y}_{\mathcal{I}_k}^{(j)} = \left(y_k^{(j)}; k \in \mathcal{I}_k\right)$.

Step 4. Finding the optimum solution with Bayesian inference

We solve the following *Optimum Design Problem with Bayesian Inference* (*ODPBI*).

<u>*ODPBI*</u>

$$\min_{\boldsymbol{x}} \quad f(\boldsymbol{x}) = \sum_{i=1}^{n_M} w_F \rho l_i A_i(\boldsymbol{x}) + \sum_{j=1}^{n_F} w_D f_{Dj}(\boldsymbol{x}) \tag{3.26}$$

$$\text{subject to} \quad \left| \mathbb{E}\left[\delta_{j\max}^V(\boldsymbol{x}) \mid \boldsymbol{y}^{(j)}, \hat{\alpha}^{(j)}, \hat{\psi}^{(j)} \right] \right| + \sqrt{\mathbb{V}\left[\delta_{j\max}^V(\boldsymbol{x}) \mid \boldsymbol{y}^{(j)}, \hat{\alpha}^{(j)}, \hat{\boldsymbol{\psi}}^{(j)} \right]} \le \overline{\delta}_{j\max} \quad (j=1,\cdots,n_F) \tag{3.27}$$

$$-\overline{\varepsilon}_{\max} \le \varepsilon_i(\boldsymbol{x}) \le \overline{\varepsilon}_{\max} \quad (i=1,\cdots,n_M) \tag{3.28}$$

$$\overline{x}_l^L \le x_l \le \overline{x}_l^U \quad (l=1,\cdots,n_X) \tag{3.29}$$

$$\hat{x}_l^0 - \Delta r_l \le x_l \le \hat{x}_l^0 + \Delta r_l \quad (l=1,\cdots,n_X) \tag{3.30}$$

The solution of *ODPBI* is denoted by $\hat{\boldsymbol{x}} = \{\hat{x}_l; l=1,\cdots,n_X\}$. *ODPBI* is based on the calibrated simple model, besides, the output of the calibration model is smoothed out, as a result solving *ODPBI* needs low computational cost and is numerically stable.

Step 5. Verification of the solution

We analyze the solution $\hat{\boldsymbol{x}}$ by verification model. If the output $\{\hat{y}^{(j)}(\hat{\boldsymbol{x}}); j=1,\cdots,n_F\}$ doesn't satisfy the design criteria with sufficient accuracy, $\{\hat{\boldsymbol{x}}, \hat{y}^{(j)}(\hat{\boldsymbol{x}})\}$ is added to the training data set $\mathcal{D}_j$ as a new data. Moreover, we repeat from step3 until the solution satisfies the design criteria.

4. NUMERICAL EXAMPLE

Consider 10-story and 3-span planar steel frames as shown in Figure 3. Cross sections of columns and beams are denoted by C1-C8 and G1-G10, respectively. The dampers of the 1-10th story are denoted by D1-D10.

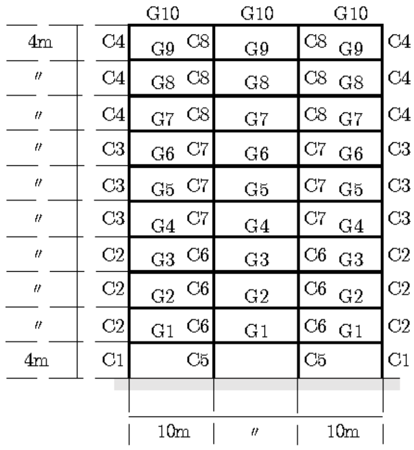

Figure 3 10-stories and 3-span model

The structural damping ratio is assumed as $h = 2\%$. A lamped mass of 326.5kg and 163.3kg are placed on every interior node and exterior node, respectively. The parameters of side constrains are as shown:

for beams: $\bar{x}_l^L = 100\text{cm}^2$, $\bar{x}_l^U = 400\text{cm}^2$, $\Delta r_l = 90\text{cm}^2$,
for columns: $\bar{x}_l^L = 200\text{cm}^2$, $\bar{x}_l^U = 800\text{cm}^2$, $\Delta r_l = 180\text{cm}^2$,
for dampers: $\bar{x}_l^L = 0\text{kN}$, $\bar{x}_l^U = 100\text{kN}$, $\Delta r_l = 30\text{kN}$.

The other parameters are as follows: $\bar{\varepsilon}_{\max} = 0.00157$, $\bar{\delta}_{j\max} = 2\text{cm}$, $w_F = 25/\text{ton}$, $w_D = 0.15/\text{kN}$. Here, practical constraint conditions are added to *SODP* and *ODPBI*. The condition is that a cross section of a column is smaller than cross sections of the lower columns ($C1 > C2 > C3 > C4$, $C5 > C6 > C7 > C8$).
Firstly, we obtain the initial solution $\hat{\boldsymbol{x}}^0$ by solving *SODP* with sequential quadratic programming (step1). Next, computer experiments, of which the number is given by $n = 100$, are carried out (step2). Lastly, we obtain the optimum solution $\hat{\boldsymbol{x}}$ by 10 iterations of solving and updating *ODPBI* with sequential quadratic programming (from step3 to step 5). The solutions are shown in Table 2. The costs and maximum story drift angles by time history response analysis of the optimum solutions $\hat{\boldsymbol{x}}^0$ and $\hat{\boldsymbol{x}}$ are shown in Table 3, where $H = 400\text{cm}$ denotes the story height. It can be observed that the costs of $\hat{\boldsymbol{x}}^0$ and $\hat{\boldsymbol{x}}$ are nearly the same. The maximum story drift angles of the solution $\hat{\boldsymbol{x}}^0$ violate the criteria $\bar{\delta}_{j\max}/H = 5/1000\,\text{rad}$ largely, by contrast, the maximum story drift angles of the solution $\hat{\boldsymbol{x}}$ approximately satisfy the criteria $5/1000\,\text{rad}$ because the predicted mean and variance have good accuracy.

5. CONCLUSION

The conclusion may be summarized as follows:
(1) We present a new optimum design method of a building frame with viscous dampers using the calibration model, which is based on the statistical multi-level analysis. The method has similar accuracy as the verification analysis and the computational cost is much smaller than the verification analysis.
(2) The efficiency of the presented method is demonstrated by numerical example. In the example, the predicted mean and variance have good accuracy. Consequently, the solution approximately satisfies the constraints.

Table 2 Optimum solutions

Cross sectional area of Colum (cm^2)	$\hat{\boldsymbol{x}}^0$	$\hat{\boldsymbol{x}}$	Cross sectional area of Beam (cm^2)	$\hat{\boldsymbol{x}}^0$	$\hat{\boldsymbol{x}}$	Maximum load of Damper (kN)	$\hat{\boldsymbol{x}}^0$	$\hat{\boldsymbol{x}}$
C1	268.9	254.7	G1	253.7	256.1	D1	100.0	70.0
C2	268.9	254.7	G2	323.4	321.6	D2	100.0	100.0
C3	217.9	209.6	G3	238.1	244.8	D3	100.0	100.0
C4	200.0	200.0	G4	281.6	310.4	D4	100.0	100.0
C5	404.2	370.7	G5	250.3	250.3	D5	100.0	100.0
C6	404.2	370.7	G6	235.8	247.1	D6	100.0	87.1
C7	380.9	352.7	G7	191.7	231.3	D7	100.0	83.9
C8	281.0	316.6	G8	248.6	201.8	D8	100.0	100.0
			G9	108.8	162.4	D9	100.0	70.0
			G10	100.0	118.2	D10	100.0	70.0

Table 3 Cost and maximum story drift angle by time history response analysis of the optimum solution

	Cost	Maximum story drift angles [1/1000 rad]	1F	2F	3F	4F	5F	6F	7F	8F	9F	10F
$\hat{\boldsymbol{x}}^0$	2381.2	$\delta^V_{\max}(\hat{\boldsymbol{x}}^0)/H$	3.00	4.16	4.32	4.42	4.74	4.39	4.64	5.20	5.09	6.59
$\hat{\boldsymbol{x}}$	2398.6	$\delta^V_{\max}(\hat{\boldsymbol{x}})/H$	**3.75**	**5.09**	**4.94**	**4.90**	**4.98**	**4.80**	**4.35**	**4.77**	**4.55**	**4.78**
		$\mathbb{E}\left[\delta^V_{\max}(\hat{\boldsymbol{x}})\mid\boldsymbol{y}\right]/H$	3.74	4.93	4.95	4.81	4.99	4.58	4.61	4.84	4.65	4.56
		$\sqrt{\mathbb{V}\left[\delta^V_{\max}(\hat{\boldsymbol{x}})\mid\boldsymbol{y}\right]}/H$	0.00	0.07	0.03	0.19	0.01	0.03	0.39	0.16	0.10	0.44
		$\left\lvert\delta^V_{\max}(\hat{\boldsymbol{x}})-\mathbb{E}\left[\delta^V_{\max}(\hat{\boldsymbol{x}})\mid\boldsymbol{y}\right]/H\right\rvert$	0.02	0.17	0.01	0.09	0.00	0.23	0.26	0.07	0.10	0.23

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REFERENCES

Igusa, T., Der Kiureghian, A. and Sackman, J. L. (1984). Modal decomposition method for stationary response of non-Classically damped systems. *Earthquake Engng. Struct. Dynamics*, 12, 121-136

Kennedy, M. C., and O'hagan, A. (2001). A Bayesian calibration of computer models (with discussion). *Journal of the Royal Statistical Society*, B, 63, 425-464

Newmark, N. M., and Hall, W. J. (1982). Earthquake spectra and design, Earthquake Engineering Research Institute, Berkeley, CA.

Rasmussen, C. E., and Williams, C. K. (2006). Gaussian Processes for Machine Learning, The MIT Press.

Santner, T. J., Williams, B. J., and Notz, W. I. (2003). The design and Analysis of Computer Experiments, Springer-Verlag.

Tsuji, M., Nagano, Y., Ohsaki, M. and Uetani, K. (2000) Optimum Design Method for High-Rise Building Frame with Viscous Dampers, *12th WCEE*, CD-ROM, New Zealand.